DAR model notes NEW

**The DAR(1) model**

**Functional Form**

with

**Conditional Variance and Expextation**

**The DAR(1)-MA(1) model**

**Functional Form**

General form,

With parameters defined for , , , and .

Using that with , the model can be reformulated as,

Now define such that the model becomes,

It is evident that the term is highly non-linear. In the following, we will assume that such that,

**Conditional Mean**

Thus,

**Conditional Variance**

The conditional variance can be decomposed as,

We start with ,

Using that and we can reduce the expression to,

Therefore,

Now for ,

Now ,

This reduces to,

**Distribution**

We have derived that is conditionally distributed as,

Assuming that , the conditional distribution of is then,

With and

As depends on both and , the model violates the Markov-chain assumption assumptin I.3.1 (i) in ARCH Part 1. However, we can reformulate the model on companion form as,

Using this formulation, conditional on the past values , depends only on as shown,

However, violates the assumptin I.3.1 (ii) in ARCH Part 1, as it is singular. This can be shown by using the formula for conditional densities , yielding,

As shown before, the first term is is continous Gaussian density. But, , since is fixed (already in the information set). Therefore, is no longer a random variable, and the conditional density function becomes a Dirac delta function at , which is not continues, violating the assumption I.3.1 (ii).

The problem of singularity can be solved by instead conditioning on ,

Since is a continious Gaussian density, it follows that is also. It holds that the product of two continous Gaussian densities is also a continous Gaussian density, therefore satisfies assumption I.3.1 (ii).

**Maximum Likelihood Estimation**

Ignoring all constants, i.e. , the Gaussian log-likelihood function for the DARMA model is defined as,

with , , and . Since the maximum lag is , we can only estimate the model using observations.

**Deriving using recursions for**

Even though is highly non-linear we can derive a nice expression using recursion for ,

Inserting ,

Respectively insert and , and reduce,

Again, respectively insert and , and reduce,

It is therefore evident that,

The DAR-MA(1) model can then be written as,

**Forecast (1-step-ahead)**

Standard representation forecast,

with and

Beta representation forecast,

**The DAR(1)-MA(1)-X model**

**Functional Form**

General form,

Volatility induced stationarity.

With parameters , , , and .